Cape Cod Community College

Department Syllabus

Prepared by the Department of Mathematics
Date of Departmental Approval: December 4, 2017
Date approved by Curriculum and Programs: January 24, 2018 Effective: Fall 2018

1. **Course Number: MAT245**
   Course Title: Linear Algebra

2. **Description:**
   A first course in the study and analysis of linear systems and their applications in mathematics, engineering, computer science, business, economics, and other fields involving large multi-variate models of real world phenomena. Topics include: matrices, determinants, vectors in 2-dimensional space and 3-dimensional space, vector spaces, independence, basis, rank, linear transformations with matrix representation, eigenvalues and eigenvectors, applications to differential equations.

3. **Student Learning Outcomes (instructional objectives: intellectual skills):**
   Upon successful completion of this course, students are able to do the following:
   - Rewrite a linear system as a vector/matrix equation and use the row reduction algorithm to analyze the solution set of a linear system.
   - Test whether a vector in R^n belongs to the null/column space of a matrix.
   - Perform the three operations in matrix algebra and recite their properties.
   - Geometrically describe the solution set of a linear system with 3 variables.
   - Find the standard matrix of a linear transformation from R^m to R^n and determine whether such transformation is one-to-one/onto.
   - Recite and justify different characterizations of invertible matrices. Find A^{-1} of an invertible matrix A algorithmically. Express A^{-1} in terms of the adjoint.
   - Compute the determinant by (a) cofactor expansion, (b) row operations.
   - Demonstrate that R^n, P_n, M_{mxn}, and C[a , b] under the usual operations satisfy all ten axioms of a vector space.
   - Determine whether a nonempty subset of a vector space forms a subspace by (a) checking closure under both operations, (b) creating a matrix with the prescribed null/column space.
   - Prove/disprove that a given set of vectors forms a basis from the definition.
   - Find a basis and state the dimension of the null/column/row space of any matrix. Use the rank theorem (when applicable) to deduce these dimensions.
   - Calculate the coordinates (relative to a given basis) of any vector in R^n / P_n and find the change of coordinate matrix from one basis in R^n / P_n to another.
   - Determine the eigenvalue for a given eigenvector of a square matrix.
   - Diagonalize a 3×3 matrix whenever it has 3 independent eigenvectors.

4. **Credits(s):** 3 credits

5. **Satisfies General Education Requirement:** No

6. **Prerequisite:** A grade of C- or higher in MAT240 (Calculus I) or MAT180 (Applied Calculus)

7. **Semester Offered:** Fall

8. **Suggested General Guidelines for Evaluation:** Comprehensive final examination, hour tests, problems, cases, and quiz papers.

9. General Topical Outline (Optional):

I. Systems of Linear Equations and Matrices
   A. Introduction to Systems of Linear Equations
   B. Gaussian Elimination
   C. Homogeneous Systems of Linear Equations
   D. Matrices and Matrix Operations
   E. Rules of Matrix Operations
   F. Rules of Matrix Arithmetic
   G. Elementary Matrices and a Method for Finding A^{-1}
   H. Further Results on Systems of Equations and Invertibility

II. Determinants
   A. The Determinant Function
   B. Evaluating Determinants by Row Reduction
   C. Properties of the Determinant Function
   D. Cofactor Expansion; Cramer's Rule

III. Vectors in 2-Space and 3-Space
   A. Introduction to Vectors (Geometric)
   B. Norm of a Vector; Vector Arithmetic
   C. Dot Product; Projections
   D. Cross Product
   E. Lines and Planes in 3-Space

IV. Vector Spaces
   A. Euclidean *n*-Space
   B. General Vector spaces
   C. Subspaces
   D. Linear Independence
   E. Basis and Dimension
   F. Row and Column Space of a Matrix; Rank; Applications to Finding Bases
   G. Inner Product Spaces
   H. Length and Angle in Inner Product spaces
   I. Orthonormal Bases; Gram-Schmidt Process
   J. Coordinates; Change of Basis

V. Linear Transformations
   A. Introduction to Linear Transformations
   B. Properties of Linear Transformations: Kernel and Range
   C. Linear Transformations for R^n to R^m; Geometry of Linear Transformations from R^2 to R^2
      1. Matrices of Linear Transformations
      2. Similarity

VI. Eigenvalues, Eigenvectors
      1. Eigenvalues and Eigenvectors
      2. Diagonalization
      3. Orthogonal Diagonalization; Symmetric Matrices

VII. Applications
      1. Application to Differential Equations